



Cambridge International AS & A Level

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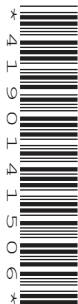
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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 Let a be a positive constant.

- (a) Use the method of differences to find $\sum_{r=1}^n \frac{1}{(ar+1)(ar+a+1)}$ in terms of n and a . [4]

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- (b) Find the value of a for which $\sum_{r=1}^{\infty} \frac{1}{(ar+1)(ar+a+1)} = \frac{1}{6}$. [3]

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2 The points A, B, C have position vectors

$$4\mathbf{i} - 4\mathbf{j} + \mathbf{k}, \quad -4\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}, \quad 4\mathbf{i} - \mathbf{j} - 2\mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

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- 3 The sequence of positive numbers u_1, u_2, u_3, \dots is such that $u_1 > 4$ and, for $n \geq 1$,

$$u_{n+1} = \frac{u_n^2 + u_n + 12}{2u_n}.$$

- (a) By considering $u_{n+1} - 4$, or otherwise, prove by mathematical induction that $u_n > 4$ for all positive integers n . [5]

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(b) Show that $u_{n+1} < u_n$ for $n \geq 1$.

[3]

4 The cubic equation $2x^3 + 5x^2 - 6 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$. [3]

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(b) Find the value of $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6}$. [3]

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- (c) Find also the value of $\frac{1}{\alpha^9} + \frac{1}{\beta^9} + \frac{1}{\gamma^9}$. [2]

(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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(d) Sketch the curve with equation $y = \left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right|$ and state the set of values of k for which $\left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right| = k$ has 4 distinct real solutions. [2]

6 The curve C has polar equation $r^2 = \tan^{-1}\left(\frac{1}{2}\theta\right)$, where $0 \leq \theta \leq 2$.

(a) Sketch C and state, in exact form, the greatest distance of a point on C from the pole. [3]

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(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = 2$. [5]

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7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

(a) Find the set of values of k for which **A** is non-singular. [3]

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(b) Given that **A** is non-singular, find, in terms of k , the entries in the top row of \mathbf{A}^{-1} . [4]

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- (d) Find the set of values of k for which the transformation in the x - y plane represented by $\begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$ has two distinct invariant lines through the origin. [6]

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